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MEMORANDUM

A GENERALIZED HYDRODYNAMIC-IMPACT THEORY FOR THE LOADS
AND MOTIONS OF DEEPLY IMMERSED PRISMATIC BODIES

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A GENERALIZED HYDRODYNAMIC-IMPACT THEORY FOR THE LOADS
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SUMMARY

A theory is derived for determining the loads and motions of a deeply immersed prismatic body. The method makes use of a two-dimensional water-mass variation and an aspect-ratio correction for three-dimensional flow. The equations of motion are generalized by using a mean value of the aspect-ratio correction and by assuming a variation of the two-dimensional water mass for the deeply immersed body. These equations lead to impact coefficients that depend on an approach parameter which, in turn, depends upon the initial trim and flight-path angles.

Comparison of experiment with theory is shown at maximum load and maximum penetration for the flat-bottom (0° dead-rise angle) model with beam-loading coefficients from 36.5 to 133.7 over a wide range of initial conditions. A dead-rise angle correction is applied and maximum-load data are compared with theory for the case of a model with 30° dead-rise angle and beam-loading coefficients from 208 to 530.

INTRODUCTION

Several theories exist for the prediction of hydrodynamic-impact loads. A survey of the earlier theories which deal primarily with vertical impacts is presented in reference 1, together with an extension to include the effect of velocity parallel to the keel. In reference 2 it is shown that this modified theory can be expressed in generalized terms. References 1 and 2 deal mainly with the impacts of lightly loaded bodies having little or no chine immersion. In reference 3 the theory is extended to include the prismatic body with immersed chines, a step-by-step procedure being utilized for the determination of the important impact parameters. It is the purpose of the present investigation to provide a generalized impact theory for the case of deeply immersed chines. The formulation of the theory is based on the assumption that

the water-mass variation throughout the impact is the same as that for the case of deeply immersed chines. Of course, this assumption is not valid when the nonimmersed portion of the impact is an important part of the total impact, and hence a deeply immersed impact, as far as this report is concerned, is one in which the nonimmersed portion plays a relatively minor role. In actuality, this condition is approached with the impact of a heavily loaded configuration, an important case since the present trend is toward such design.

In the present analysis the equations of motion are written in general terms leading to expressions that are applicable to either immersed or nonimmersed cases. The expressions are applied to the deeply immersed case by the substitution of an assumed two-dimensional water-mass variation, resulting in relations that allow such parameters as acceleration, penetration, and time to be written in a coefficient form that is dependent only upon the velocity ratio and an approach parameter. This generalization reduces considerably the number of solutions necessary to cover the entire range of impact conditions and also simplifies the presentation of experimental results.

Theoretical and experimental curves of the acceleration and penetration coefficients are presented at maximum load and maximum penetration for the case of 0° dead-rise angle over a wide range of initial conditions for bodies having beam-loading coefficients ranging from 36.5 to 133.7. For bodies having a 30° dead-rise angle and beam-loading coefficients from 208 to 530, a correcting factor is applied and experimental maximum-load data are compared with the theory. Also for the case of 0° dead-rise angle with beam-loading coefficients of 18.8 and 4.4, correction factors are applied to experimental results and these results are compared with the present theory for the maximum acceleration coefficient.

SYMBOLS

b	beam
F	hydrodynamic force
g	acceleration due to gravity
G	constant (eq. (18))
l	wetted length along keel
m_w	two-dimensional water mass in transverse plane

s	distance along keel from foremost immersed station to flow plane
\dot{s}	velocity along keel
t	time after water contact
V	resultant velocity
W	weight
x	distance parallel to undisturbed water surface, positive in direction of body motion
\dot{x}	horizontal velocity
\ddot{x}	horizontal acceleration
z	immersion of keel at step, measured normal to undisturbed water surface, positive downward
\dot{z}	vertical velocity
\ddot{z}	vertical acceleration
β	angle of dead rise
γ	flight-path angle
ξ	distance from undisturbed water surface to keel in any given flow plane, measured normal to keel, positive downward
$\dot{\xi}$	velocity normal to keel
$\ddot{\xi}$	acceleration normal to keel
λ	ratio of length of keel below undisturbed water surface to the beam
ρ	mass density
τ	trim angle
$\phi(\lambda)$	aspect-ratio correction
$\phi(\lambda_1)$	mean aspect-ratio correction

Subscripts:

o	at water contact
s	at step
x	horizontal direction
z	vertical direction
ζ	normal to keel
max	maximum

Dimensionless parameters:

C_{Δ}	beam-loading coefficient, $\frac{W}{\rho g b^3}$
κ	approach parameter, $\frac{\sin \tau}{\sin \gamma_0} \cos(\tau + \gamma_0)$
C_a	acceleration coefficient, $-\frac{\ddot{z}b}{\dot{z}_0^2} \left(\frac{2W \sin \tau \cos \tau}{G \phi(\lambda_1) \rho g b^3} \right)^{1/2}$
C_d	penetration coefficient, $\frac{z}{b} \left(\frac{G \phi(\lambda_1) \rho g b^3}{2W \sin \tau \cos \tau} \right)^{1/2}$
C_t	time coefficient, $\frac{t \dot{z}_0}{b} \left(\frac{G \phi(\lambda_1) \rho g b^3}{2W \sin \tau \cos \tau} \right)^{1/2}$
\dot{z}/\dot{z}_0	vertical-velocity ratio

THEORY

General

The basic theory follows closely that of references 1 to 3. It is based on the idea that the flow about a slender immersing body occurs in transverse planes oriented perpendicular to the keel. The motion of the fluid in each plane is treated as a two-dimensional phenomenon and an aspect-ratio correction is applied for the three-dimensional case. The

effects of buoyancy, viscosity, and trim changes are neglected and the reaction in a given flow plane ds (fig. 1) is written as

$$dF = \frac{\partial}{\partial t} (m_w \dot{\xi}) ds \quad (1)$$

$$dF = \left(\dot{\xi} \frac{\partial m_w}{\partial t} + m_w \ddot{\xi} \right) ds \quad (2)$$

By integrating equation (2) over the wetted length l the expression for the hydrodynamic force is given as

$$F = \int_0^l \dot{\xi} \frac{\partial m_w}{\partial t} ds + \int_0^l m_w \ddot{\xi} ds \quad (3)$$

By using Newton's second law and an aspect-ratio correction $\varphi(\lambda)$ for three-dimensional flow, equation (3) may be written

$$- \frac{W}{g} \ddot{\xi} = \varphi(\lambda) \left(\int_0^l \dot{\xi} \frac{\partial m_w}{\partial t} ds + \int_0^l m_w \ddot{\xi} ds \right) \quad (4)$$

Since

$$\frac{\partial m_w}{\partial t} = \frac{\partial m_w}{\partial \xi} \frac{\partial \xi}{\partial t} = \frac{dm_w}{d\xi} \dot{\xi} \quad (5)$$

and

$$d\xi = \tan \tau ds \quad (6)$$

the first term on the right-hand side of equation (4) may be integrated and the results rearranged to give

$$\ddot{\xi} \left[1 + \frac{g \varphi(\lambda)}{W} \int_0^l m_w ds \right] = - \frac{g \varphi(\lambda)}{W \tan \tau} m_{w,s} \dot{\xi}^2 \quad (7)$$

where $m_{w,s}$ is the two-dimensional water mass evaluated at the step.

To obtain the equation of motion (7) in terms of the vertical components, the following substitutions are made:

$$\ddot{\xi} = \frac{\ddot{z}}{\cos \tau} \quad (8)$$

and

$$\dot{\xi} = \frac{\dot{z}}{\cos \tau} - \dot{s} \tan \tau \quad (9)$$

Equation (7) may then be written

$$\frac{\ddot{z}}{(\dot{z} - \dot{s} \sin \tau)^2} = - \frac{\frac{m_{w,s}}{\sin \tau}}{\frac{W}{g \varphi(\lambda)} + \int_0^l m_w ds} \quad (10)$$

Multiplying equation (10) by dz , substituting $\ddot{z} dz = \dot{z} d\dot{z}$, and integrating gives

$$\int_{\dot{z}_0}^{\dot{z}} \frac{\dot{z} d\dot{z}}{(\dot{z} - \dot{s} \sin \tau)^2} = - \int_0^z \frac{\frac{m_{w,s}}{\sin \tau} dz}{\frac{W}{g \varphi(\lambda)} + \int_0^l m_w ds} \quad (11)$$

or

$$\log \frac{\frac{\dot{z}}{\dot{z}_0} + \kappa}{1 + \kappa} + \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} - \frac{\kappa}{1 + \kappa} = - \int_0^z \frac{\frac{m_{w,s}}{\sin \tau} dz}{\frac{W}{g \varphi(\lambda)} + \int_0^l m_w ds} \quad (12)$$

where $\kappa = - \frac{\dot{s} \sin \tau}{\dot{z}_0} = \frac{\sin \tau}{\sin \gamma_0} \cos(\tau + \gamma_0)$ and its variation with trim

and flight-path angles is shown on figure 2. Equation (12) is general in that it may be applied to impacts with chines either immersed or non-immersed upon proper substitution of the two-dimensional water-mass variation and the aspect-ratio correction. If, for instance, the water-mass variation is considered proportional to the penetration squared and the aspect-ratio correction is taken as a constant, the equation

can be integrated directly, leading to the equations of reference 2. In reference 3 an equation similar to equation (12) is derived and applied to the immersed-chine case. In the application the term $\int_0^l m_w ds$ is eventually neglected and the equations of motion are solved by a step-by-step procedure involving the Pabst aspect-ratio correction and various water-mass variations throughout the impact.

In equation (12), if it is assumed that $\varphi(\lambda)$ can be approximated by a mean value and taken as a constant with respect to the integration, the expression may be integrated directly. The right-hand side of equation (12) is of the $\frac{du}{u}$ form since

$$\frac{d}{dz} \left(\frac{W}{g \varphi(\lambda_1)} + \int_0^l m_w ds \right) = \frac{dl}{dz} m_{w,s} = \frac{m_{w,s}}{\sin \tau}$$

where $\varphi(\lambda_1)$ is a representative mean value for the aspect-ratio correction. Carrying out the integration of equation (12) leads to

$$\log \frac{\frac{\dot{z}}{\dot{z}_0} + \kappa}{1 + \kappa} + \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} - \frac{\kappa}{1 + \kappa} = \left[-\log \left(\frac{W}{g \varphi(\lambda_1)} + \int_0^z \frac{m_{w,s}}{\sin \tau} dz \right) \right]_0^z \quad (13)$$

Evaluation of the right-hand side of equation (13) gives

$$\log \frac{\frac{\dot{z}}{\dot{z}_0} + \kappa}{1 + \kappa} + \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} - \frac{\kappa}{1 + \kappa} = -\log \left[1 + \frac{g \varphi(\lambda_1)}{W} \int_0^z \frac{m_{w,s}}{\sin \tau} dz \right] \quad (14)$$

which can be written

$$\frac{g \varphi(\lambda_1)}{W} \int_0^z \frac{m_{w,s}}{\sin \tau} dz = \frac{1 + \kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} e^{\frac{\kappa}{1 + \kappa} - \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa}} - 1 \quad (15)$$

Equation (15) relates the penetration to the velocity ratio.

An expression relating the acceleration, velocity, and penetration can be obtained by dividing equation (10) by \dot{z}_0^2 and solving for $\frac{\ddot{z}}{\dot{z}_0^2}$.

The result is

$$\frac{\ddot{z}}{\dot{z}_0^2} = - \frac{\left(\frac{\dot{z}}{\dot{z}_0} + \kappa\right)^2 \frac{g \varphi(\lambda_1) m_{w,s}}{W \sin \tau}}{1 + \frac{g \varphi(\lambda_1)}{W} \int_0^z \frac{m_{w,s}}{\sin \tau} dz} \quad (16)$$

If the mean value of the aspect-ratio correction can be successfully approximated and expressions for the two-dimensional water mass are available, equations (15) and (16) can be applied in general to get the loads and motions of the hydrodynamic impact. It is interesting to note that $\varphi(\lambda)$ is taken as a constant for the nonimmersed-chine case of reference 2 and it also approaches the constant 1 for the deeply-immersed-chine case to be treated.

Deeply Immersed Chines

In the application of equations (15) and (16) to deeply immersed chines, it is necessary to select a variation of the two-dimensional water mass $m_{w,s}$. In reference 3 the variation suggested is

$$m_{w,s} = \rho b^2 \left[\frac{\pi}{8} (f(\beta) \tan \beta)^2 + \frac{B}{2} \left(\frac{\zeta'_s}{b} - \frac{\tan \beta}{2} \right) \right] \quad \left(\beta > 0^\circ; \frac{\zeta'_s}{b} > \frac{\tan \beta}{2} \right) \quad (17)$$

where ζ'_s represents the normal penetration at the step, including the effect of a water rise, and B is a theoretical constant which varies with dead-rise angle sometimes called the Bobyleff or Kirchhoff coefficient.

In the present investigation, as an approximation the nonimmersed-chine portion of the impact is disregarded and the expression for the two-dimensional water mass is written simply

$$m_{w,s} = \rho b^2 G \frac{\zeta_s}{b} = \rho b^2 G \frac{z}{b \cos \tau} \quad (18)$$

It may be noted that this expression is based upon the penetration measured from the undisturbed water surface and thus neglects the effect of water rise.

Substituting equation (18) into equation (15) and integrating gives

$$\frac{G \varphi(\lambda_1) \rho g b z^2}{2W \sin \tau \cos \tau} = \frac{1 + \kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} e^{\frac{\kappa}{1+\kappa} - \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa}} - 1 \quad (19)$$

Similarly, equation (16) becomes

$$\frac{\ddot{z}}{\dot{z}_0^2} = \frac{-2\left(\frac{\dot{z}}{\dot{z}_0} + \kappa\right)^2 \frac{G \varphi(\lambda_1) \rho g b z}{2W \sin \tau \cos \tau}}{1 + \frac{G \varphi(\lambda_1) \rho g b z^2}{2W \sin \tau \cos \tau}} \quad (20)$$

Generalized parameters.— Equations (19) and (20) are similar in form to those derived in reference 2 for the nonimmersed-chine case. As in reference 2, the equations are written in terms of generalized parameters. If, for instance,

$$C_d = \frac{z}{b} \left(\frac{G \varphi(\lambda_1) \rho g b^3}{2W \sin \tau \cos \tau} \right)^{1/2} \quad (21)$$

then from equation (19)

$$C_d = \left(\frac{1 + \kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} e^{\frac{\kappa}{1+\kappa} - \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa}} - 1 \right)^{1/2} \quad (22)$$

and if an acceleration coefficient is defined as

$$C_a = - \frac{\ddot{z} b}{\dot{z}_0^2} \left(\frac{2W \sin \tau \cos \tau}{G \varphi(\lambda_1) \rho g b^3} \right)^{1/2} \quad (23)$$

then equation (20) may be written

$$C_a = 2 \left(\frac{\dot{z}}{\dot{z}_0} + \kappa \right)^2 \frac{C_d}{1 + C_d^2} \quad (24)$$

or, in terms of velocity ratio and κ ,

$$C_a = 2 \left(\frac{\dot{z}}{\dot{z}_0} + \kappa \right)^2 \left[\frac{\frac{\dot{z}}{\dot{z}_0} + \kappa}{1 + \kappa} e^{\frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa}} - \frac{\kappa}{1 + \kappa} \left(1 - \frac{\frac{\dot{z}}{\dot{z}_0} + \kappa}{1 + \kappa} e^{\frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa}} \right) \right]^{1/2} \quad (25)$$

A time coefficient C_t may also be defined as

$$C_t = \frac{t \dot{z}_0}{b} \left(\frac{G \varphi(\lambda_1) \rho g b^3}{2W \sin \tau \cos \tau} \right)^{1/2} = \int_0^{C_d} \frac{dC_d}{\frac{\dot{z}}{\dot{z}_0}} \quad (26)$$

or

$$C_t = - \int_1^{\dot{z}/\dot{z}_0} \frac{1}{C_a} d \frac{\dot{z}}{\dot{z}_0} \quad (27)$$

Equations (21) to (27) are the generalized equations of motion for the deeply immersed impact. Writing the generalized coefficients in terms of the approach parameter κ and the vertical-velocity ratio $\frac{\dot{z}}{\dot{z}_0}$, as indicated, reduces the number of parameters necessary to describe an impact.

Maximum acceleration.— To obtain the conditions at maximum acceleration, equation (20) may be written as

$$(1 + Kz^2)\ddot{z} + 2Kz(\dot{z} + \kappa\dot{z}_0)^2 = 0 \quad (28)$$

where

$$K = \frac{G \varphi(\lambda_1) \rho g b}{2W \sin \tau \cos \tau}$$

Equation (28) can then be differentiated to give

$$\ddot{z}(1 + Kz^2) + \dot{z}\ddot{z}(2Kz) + 4Kz\dot{z}\ddot{z}(\dot{z} + \kappa\dot{z}_0) + 2K\dot{z}(\dot{z} + \kappa\dot{z}_0)^2 = 0 \quad (29)$$

At maximum acceleration the differential of the acceleration with respect to time, \ddot{z} , is equal to zero; therefore,

$$2Kz\dot{z}\ddot{z} + 4Kz\ddot{z}(\dot{z} + \kappa\dot{z}_0) + 2K\dot{z}(\dot{z} + \kappa\dot{z}_0)^2 = 0 \quad (30)$$

or

$$-\frac{\ddot{z}}{\dot{z}_0^2} = \frac{\frac{\dot{z}}{\dot{z}_0} \left(\frac{\dot{z}}{\dot{z}_0} + \kappa \right)^2}{z \left[\frac{\dot{z}}{\dot{z}_0} + 2 \left(\frac{\dot{z}}{\dot{z}_0} + \kappa \right) \right]} \quad (31)$$

Equation (31) gives a relationship for the velocity, penetration, and acceleration at maximum acceleration. If equation (31) is written in coefficient form and combined with equation (24) to eliminate the acceleration coefficient, the following expression is obtained for the penetration coefficient at maximum acceleration:

$$C_d = \left(\frac{\frac{\dot{z}}{\dot{z}_0}}{5 \frac{\dot{z}}{\dot{z}_0} + 4\kappa} \right)^{1/2} \quad (32)$$

In a similar fashion equation (32) can be used with equation (31) to give for the acceleration coefficient:

$$C_{a,\max} = \frac{\frac{\dot{z}}{\dot{z}_0} \left(\frac{\dot{z}}{\dot{z}_0} + \kappa \right)^2}{\left(\frac{\frac{\dot{z}}{\dot{z}_0}}{5 \frac{\dot{z}}{\dot{z}_0} + 4\kappa} \right)^{1/2} \left(3 \frac{\dot{z}}{\dot{z}_0} + 2\kappa \right)} \quad (33)$$

Equations (32) and (33) can be combined to give the relationship between the penetration and acceleration coefficients as

$$C_{a,\max} = \frac{2\kappa^2 C_d \left(\frac{1 - C_d^2}{1 - 5C_d^2} \right)^2}{1 + C_d^2} \quad (34)$$

The relation between κ and $\frac{\dot{z}}{\dot{z}_0}$ at maximum acceleration is obtainable from equations (32) and (22). Combining these two equations to eliminate C_d leads to

$$\log \frac{1}{1 + \kappa} + \log \frac{\left(\frac{\dot{z}}{\dot{z}_0} + \kappa\right) \left(6 \frac{\dot{z}}{\dot{z}_0} + 4\kappa\right)}{5 \frac{\dot{z}}{\dot{z}_0} + 4\kappa} = \frac{\kappa}{1 + \kappa} - \frac{\kappa}{\frac{\dot{z}}{\dot{z}_0} + \kappa} \quad (35)$$

In order to obtain the various generalized coefficients as a function of κ at maximum acceleration, it is necessary to make a trial-and-error solution of equation (35) and then eliminate the velocity ratio from the expressions for the generalized coefficients. However, this need be done only once.

Maximum penetration.— The expressions that apply at maximum penetration are easily obtained since at this time the vertical velocity ratio is equal to zero. Substituting this value into equation (22) gives for the penetration coefficient:

$$C_{d,max} = \left(\frac{1 + \kappa}{\kappa} e^{-\frac{1}{1+\kappa}} - 1 \right)^{1/2} \quad (36)$$

and for the acceleration coefficient:

$$C_a = 2\kappa^2 \left[\frac{\kappa}{1 + \kappa} e^{\frac{1}{1+\kappa}} \left(1 - \frac{\kappa}{1 + \kappa} e^{\frac{1}{1+\kappa}} \right) \right]^{1/2} \quad (37)$$

Limiting conditions.— Since the approach parameter κ may range between 0 and ∞ , it is of interest to investigate the values of the coefficients at these two conditions. In the former case the flight path at initial contact is normal to the keel and the momentum of the body is absorbed entirely by the flow planes normal to it. For this condition equation (22) for the penetration coefficient reduces to

$$C_d = \left(\frac{1 - \frac{z}{\dot{z}_0}}{\frac{\dot{z}}{\dot{z}_0}} \right)^{1/2} \quad (38)$$

From equation (25) the expression for the acceleration is found to be

$$C_a = 2 \left(\frac{\dot{z}}{\dot{z}_0} \right)^{5/2} \left(1 - \frac{\dot{z}}{\dot{z}_0} \right)^{1/2} \quad (39)$$

Combining equations (38) and (39) gives the relationship between the acceleration and penetration coefficients:

$$C_a = \frac{2C_d}{(1 + C_d^2)^{3/2}} \quad (40)$$

For the condition of $\kappa = 0$ a time coefficient can also be found directly by solving equation (38) for $\frac{\dot{z}}{\dot{z}_0}$, substituting into equation (26), and integrating. This gives

$$C_t = C_d \left(1 + \frac{1}{3} C_d^2 \right) \quad (41)$$

or, in terms of the vertical velocity,

$$C_t = \left(\frac{1 - \frac{\dot{z}}{\dot{z}_0}}{\frac{\dot{z}}{\dot{z}_0}} \right)^{1/2} \left(1 + \frac{1 - \frac{\dot{z}}{\dot{z}_0}}{3 \frac{\dot{z}}{\dot{z}_0}} \right) \quad (42)$$

The equations for the various coefficients at maximum acceleration can also be determined for the case of $\kappa = 0$. From equation (32) it is found that

$$C_d^2 = \frac{1}{5} = 0.2$$

This value represents the ratio of the virtual mass to the total mass of the body. For the nonimmersed-chine case of reference 2, a value of $2/7$ or 0.286 was obtained for this ratio. Using the value of $C_d^2 = 0.2$ in conjunction with equation (38) gives the value of $\frac{\dot{z}}{\dot{z}_0}$ at maximum acceleration:

$$\frac{\dot{z}}{\dot{z}_0} = \frac{5}{6} = 0.834$$

Reference 2 gives a value of $7/9 = 0.778$ for the nonimmersed-chine case. The other coefficients may be evaluated as follows:

$$C_d = \sqrt{0.2} = 0.448$$

$$C_a = \frac{2(0.448)}{(1 + 0.2)^3} = 0.519$$

$$C_t = (0.2)^{1/2} \left(1 + \frac{0.2}{3}\right) = 0.478$$

From equation (36) it can be seen that the maximum penetration corresponding to the case of $\kappa = 0$ is never reached as a consequence of the neglect of the buoyant forces in the theory. This may be noted also from equation (38), since letting the vertical-velocity ratio approach zero gives a value of the penetration coefficient approaching ∞ .

As κ approaches infinity the other end condition is approached, that of pure planing. In this instance, the coefficients approach the following values:

$$\lim_{\kappa \rightarrow \infty} C_d = 0$$

$$\lim_{\kappa \rightarrow \infty} \frac{\dot{z}}{\dot{z}_0} = 1$$

$$\lim_{\kappa \rightarrow \infty} C_a = \infty$$

$$\lim_{\kappa \rightarrow \infty} C_t = 0$$

These results are for the case in which the wing lift is equal to the weight.

For the planing condition with partial wing lift the relationship among the variables may be calculated by setting the left-hand side of equation (4) equal to the load on the water, letting $\zeta = 0$, and proceeding as previously. It should also be noted that $\dot{z} = 0$ for this case. Substituting the above conditions leads to the following result:

$$\frac{z}{b} = \frac{C_{\Delta,p}}{C_V^2 G \varphi(\lambda) \sin \tau}$$

where

$C_{\Delta,p}$ planing beam-loading coefficient, $L/\rho g b^3$

L load on the water

C_V speed coefficient, \dot{x}/\sqrt{gb}

DISCUSSION

The basic theory is believed to be adequate over the entire range of initial flight conditions, provided the proper two-dimensional water-mass variation is used. Since the water-mass variation of the present application is for deeply immersed chines, the application is limited to heavily loaded models having substantial chine immersion. Impacts with nonimmersed or moderately immersed chines have been analyzed in references 1 to 3.

In application of the present theory, it can be seen from equations (21) and (23) that the value of $G \varphi(\lambda_1)$ must be obtained in order to evaluate the coefficients from experimental data. This value depends upon the depth of immersion at the point of interest and the dead-rise angle. In the present application it is approximated by a constant. This approximation seems logical since the aspect-ratio correction for deep immersions asymptotically approaches 1. The value used to reduce the data of the present investigation was empirically determined for best fit; for the flat-bottom model having a range of C_{Δ} from 36.5 to 133.7 the value of $G \varphi(\lambda_1)$ was found to be approximately 1. The data of references 4 and 5 obtained from tests at the Langley impact basin were used for comparison with the theory. These data covered a range of beam-loading coefficients C_{Δ} from 36.5 to 133.7 for the model with a flat bottom.

The agreement between the theory and experiment for the acceleration coefficient is shown in figures 3 to 5. Figures 3 and 4 show the variation of the acceleration coefficient with the approach parameter at maximum load and maximum penetration, respectively, and figure 5 shows the variation of the acceleration coefficient with time. These figures show fairly good agreement between the experimental results and the theory.

The penetration coefficient at maximum penetration and maximum load is shown in figures 6 and 7, respectively. In general, the penetration is overestimated by theory, especially at the higher values of κ .

Considerable data for the body with a dead-rise angle of 30° and a C_Δ range from 208 to 503 have been obtained at the Langley impact basin (ref. 4). The trim angles ranged from 6° to 30° and the flight-path angles from approximately 2° to 20° . These data are compared with the present theory by using an empirically determined dead-rise factor to correct for the dead-rise effect. That is, the value for $G \phi(\lambda_1)$ was taken as 0.61 instead of the 1 used for the case of 0° dead-rise angle. The results are shown in figure 8 for the maximum acceleration coefficient, and the agreement suggests that the empirically determined dead-rise constant is sufficient for correcting the data at the dead-rise angle of 30° and high values of beam-loading coefficient. As a first approximation, a linear variation of $G \phi(\lambda_1)$ with dead-rise angle could be assumed to determine the intermediate values at these high beam-loading coefficients. However, more data are necessary to establish a function that will hold for all dead-rise angles.

To see how far the present application of the theory could be extended to lower beam-loading coefficients, the data of references 4 and 5 were used. These data were for flat-bottom models with C_Δ equal to 4.4 and 18.8, flight-path angles from 1.79 to 21.21, and trim angles from 3° to 45° . It was found that fair agreement could be obtained for the maximum acceleration coefficient by again merely using a different constant for the value of $G \phi(\lambda_1)$ with each of the lower beam loadings. Figures 9 and 10 show the agreement for the beam-loading coefficients of 18.8 and 4.4 when $G \phi(\lambda_1)$ is taken as 2 and 4, respectively. It is difficult to visualize any increase in $\phi(\lambda_1)$ over the value of 1 in any case; therefore, it appears that as C_Δ decreases to low values, G is dependent on C_Δ as well as on β . Of course, as C_Δ approaches 1 the theory as presently applied would be expected to show errors because of the assumption of a two-dimensional water mass.

CONCLUDING REMARKS

A theory has been derived for the loads and motions of a deeply immersed prismatic body throughout a hydrodynamic impact. The time and motion coefficients are presented in a generalized form, similar to that previously employed for the nonimmersed-chine case, which involves the use of an approach parameter κ that depends only on the initial trim and flight-path angles. The use of this parameter reduces the number of independent variables and thereby simplifies presentation of results.

The theory is substantiated over a wide range of initial flight conditions for bodies having a dead-rise angle of 0° and beam-loading coefficients C_Δ that range from 36.5 to 133.7. An empirically determined factor was used in comparing the theoretical and experimental maximum impact loads for the body with a 30° dead-rise angle and beam-loading coefficients ranging from 208 to 530. The comparison showed good agreement. Fairly good agreement was also obtained for the maximum-load data of bodies with 0° dead-rise angle and beam-loading coefficients of 18.8 and 4.4 by application of empirically determined correction factors.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., November 4, 1958.

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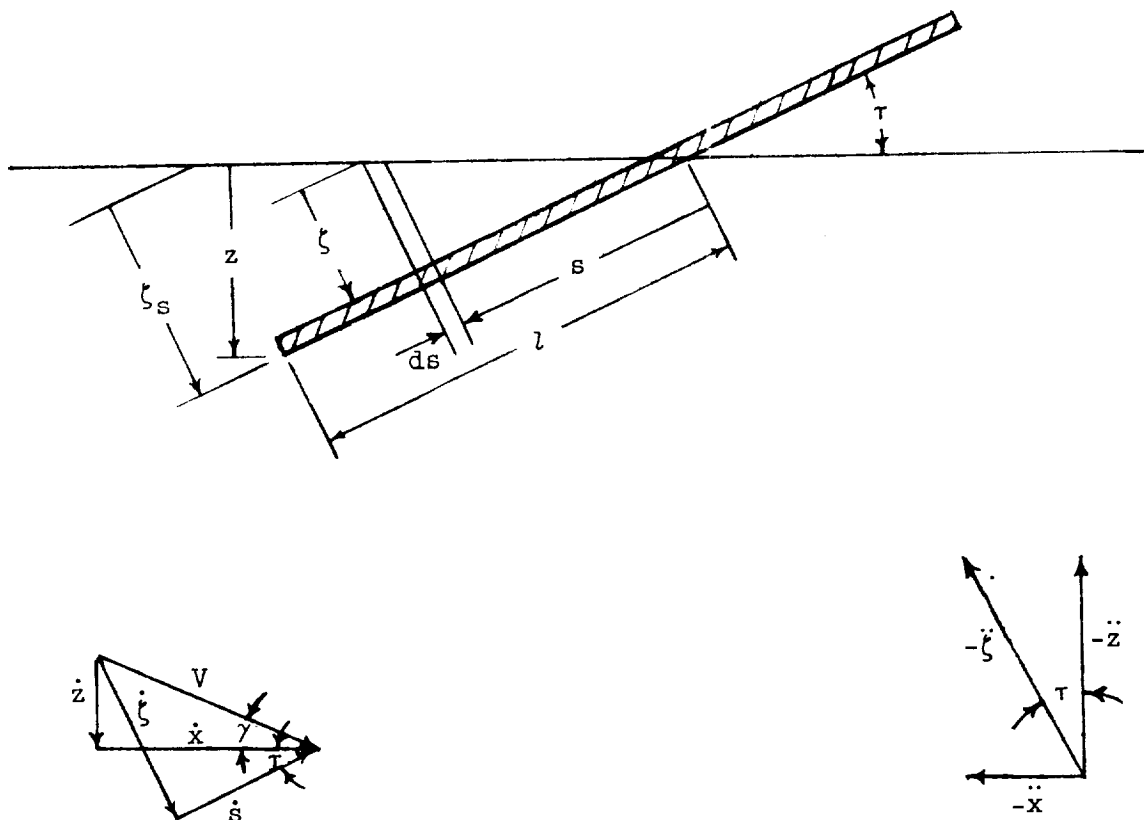


Figure 1.- Relations during impact.

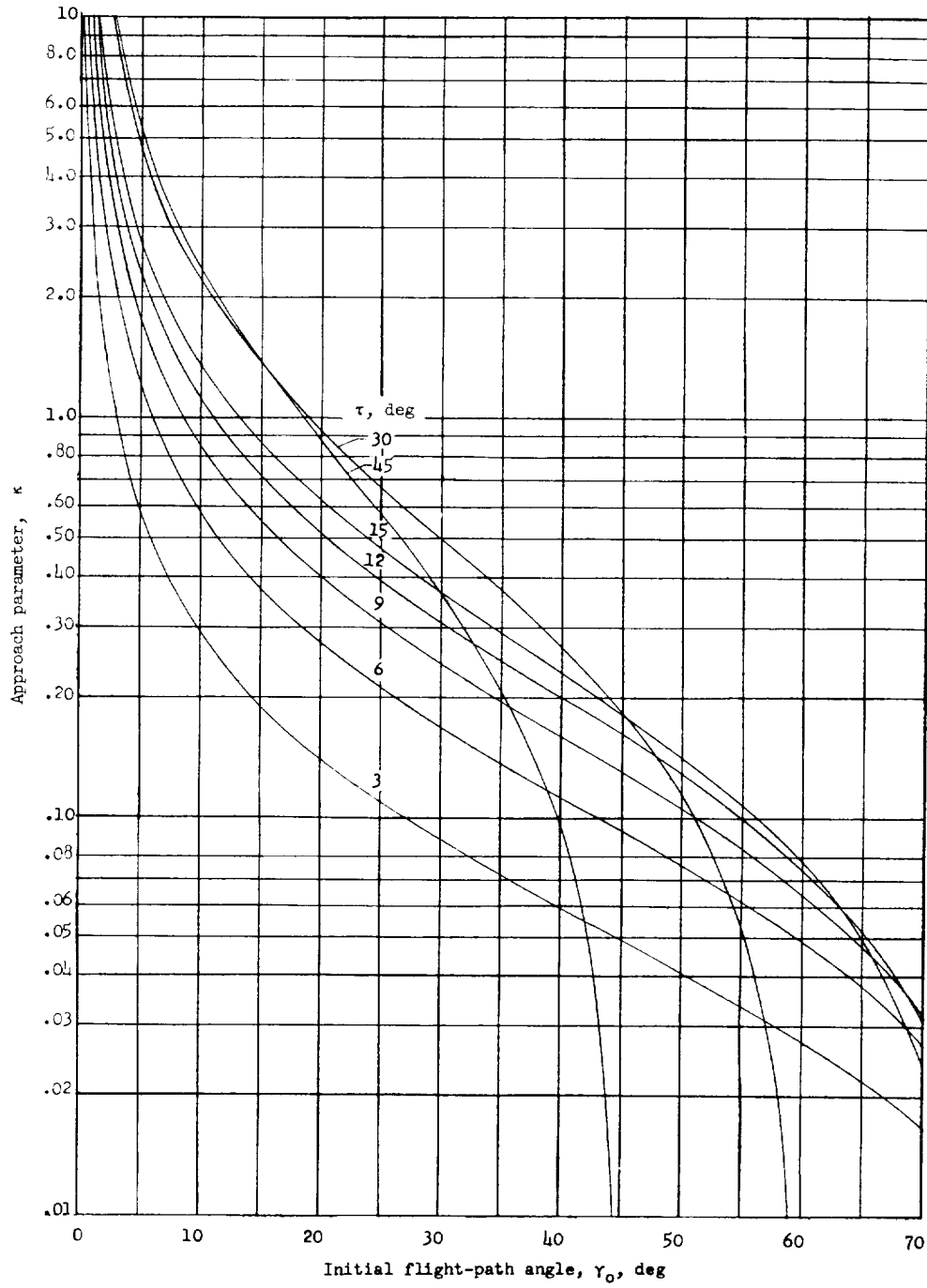


Figure 2.- Variation of approach parameter with flight-path angle at var-

ious trim angles. $\kappa = \frac{\sin \tau}{\sin \gamma_0} \cos(\tau + \gamma_0).$

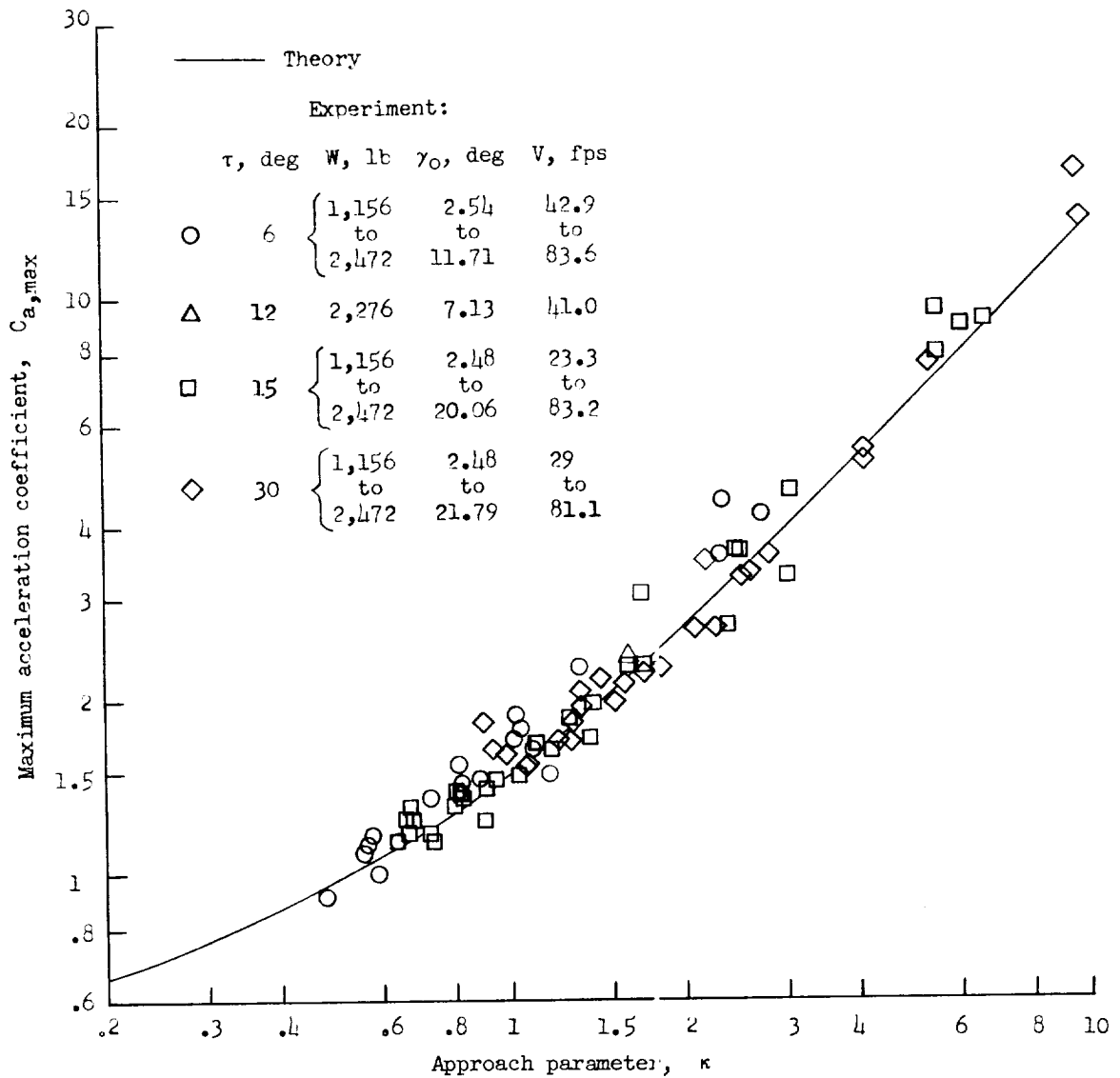


Figure 3.- Theoretical and experimental variation of maximum acceleration coefficient with approach parameter. $\beta = 0^\circ$; $b = 8$ to 12 inches; $C_\Delta = 36.5$ to 133.7.

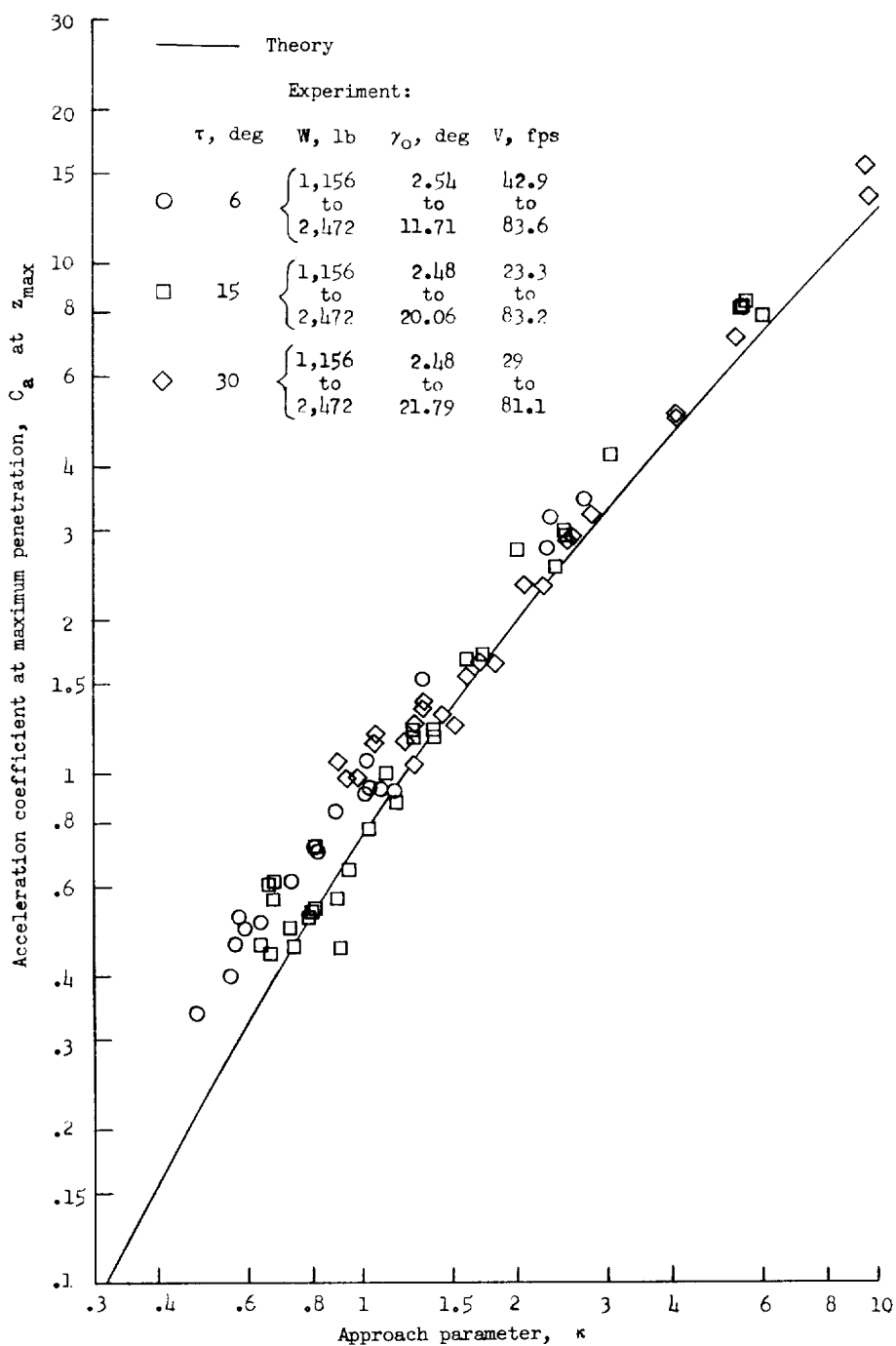


Figure 4.- Theoretical and experimental variation of acceleration coefficient at maximum penetration with approach parameter. $\beta = 0^\circ$; $b = 8$ to 12 inches; $C_\Delta = 36.5$ to 133.7.

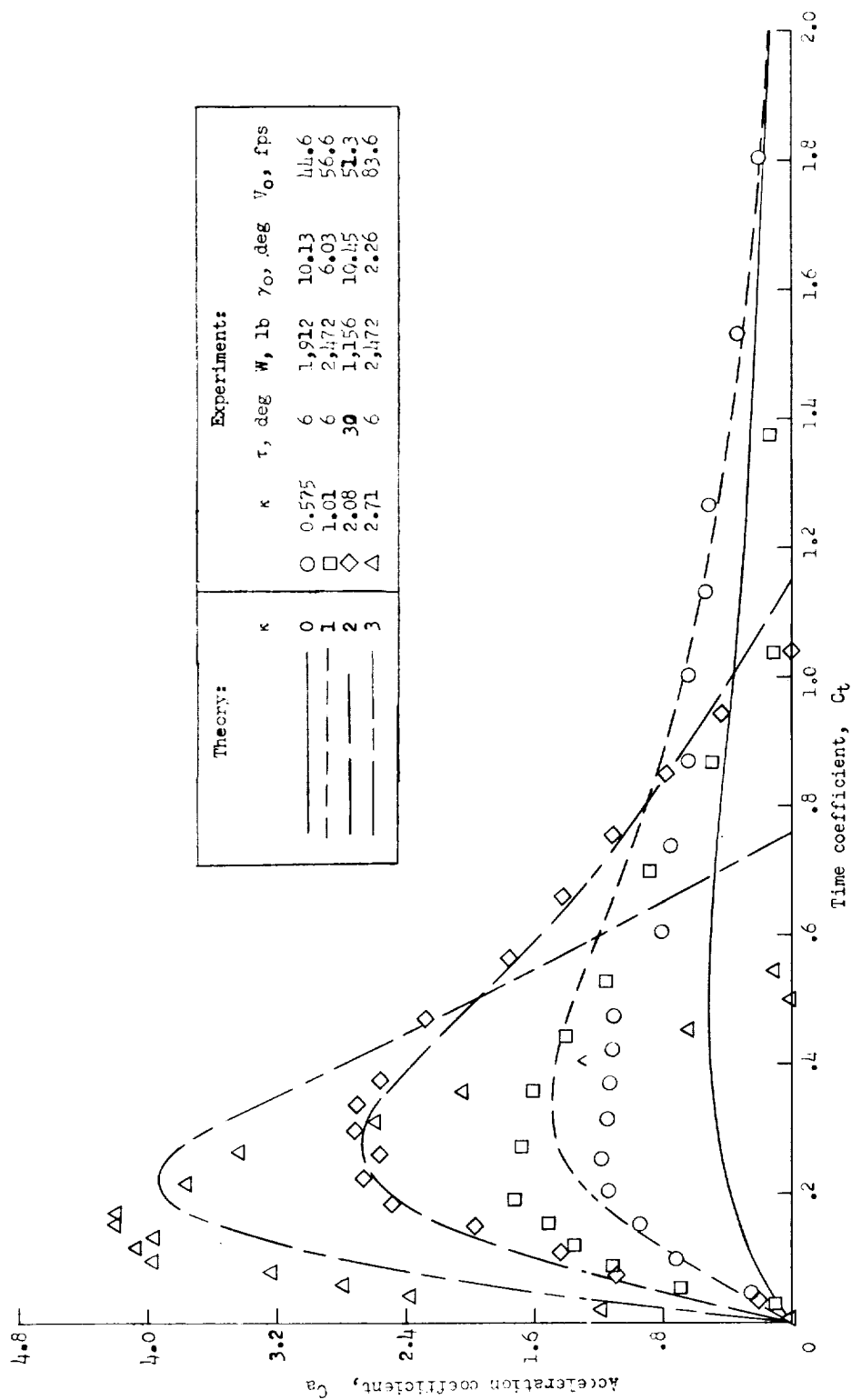


Figure 5.- Theoretical and experimental variation of acceleration coefficient with time coefficient. $\beta = 0^\circ$; $b = 8$ inches.

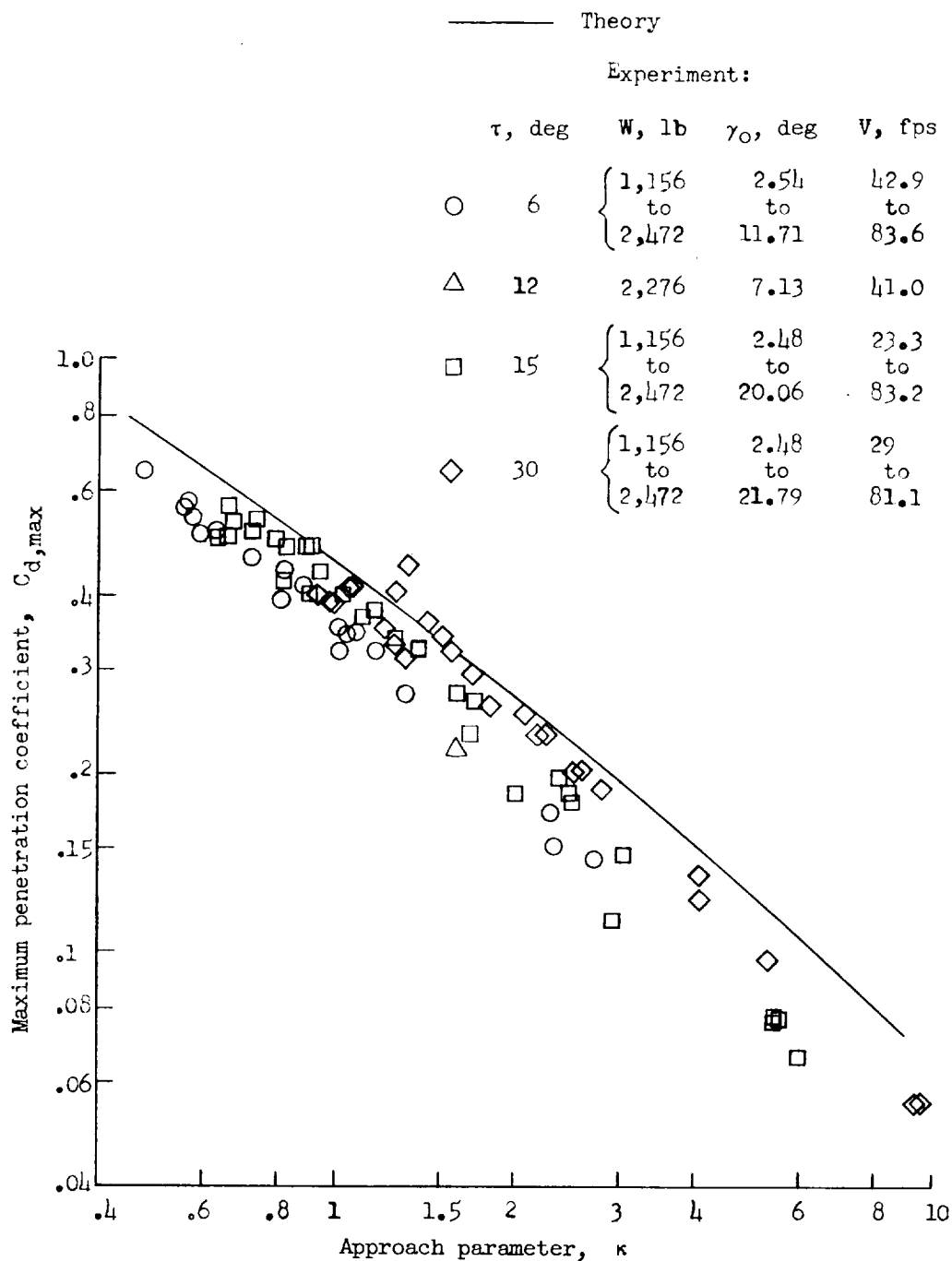


Figure 6.- Theoretical and experimental variation of maximum penetration coefficient with approach parameter. $\beta = 0^\circ$; $b = 8$ to 12 inches; $C_\Delta = 36.5$ to 133.7.

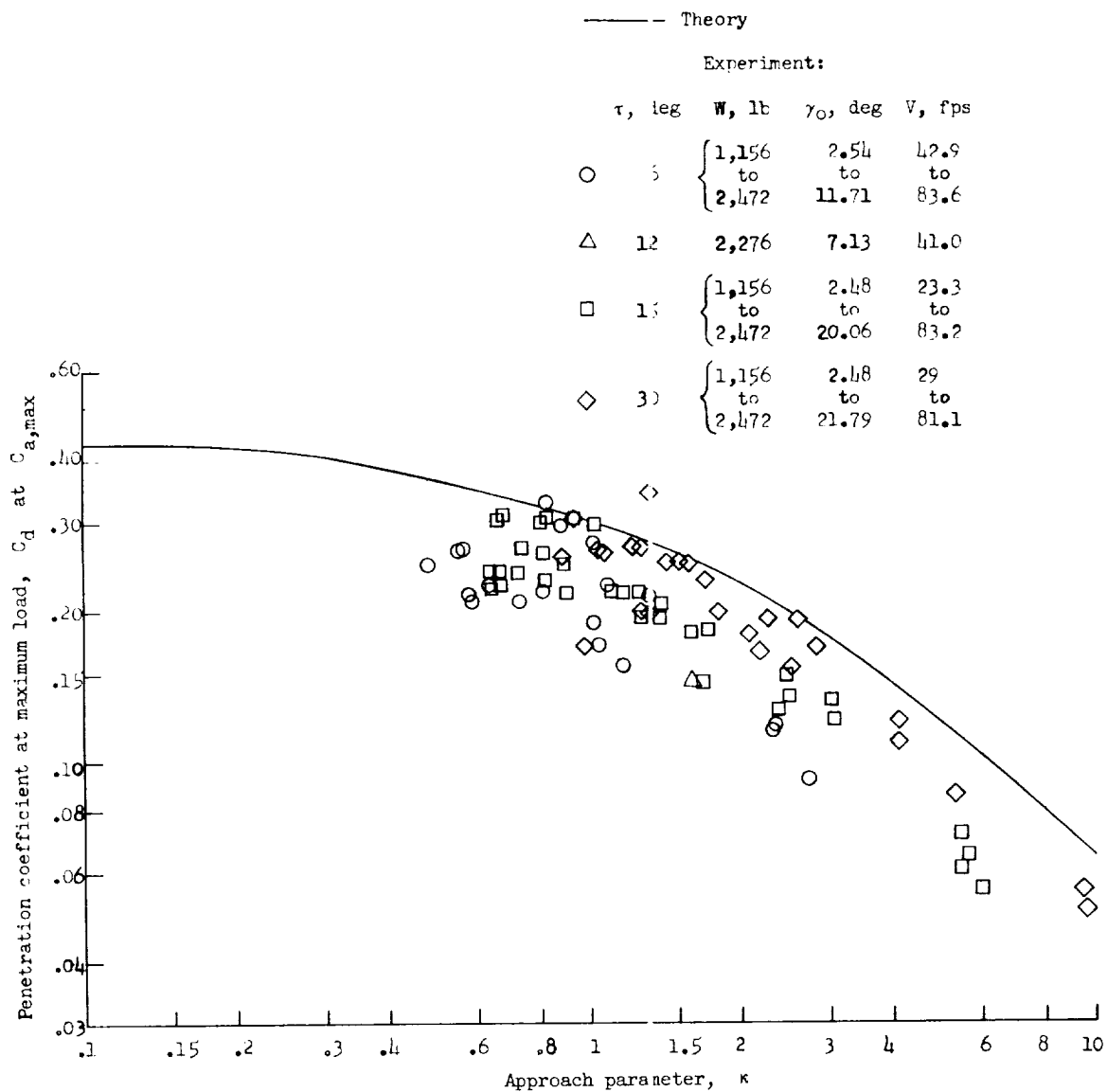


Figure 7.- Theoretical and experimental variation of penetration coefficient at maximum load with approach parameter. $\beta = 0^\circ$; $b = 8$ to 12 inches; $C_\Delta = 36.5$ to 133.7.

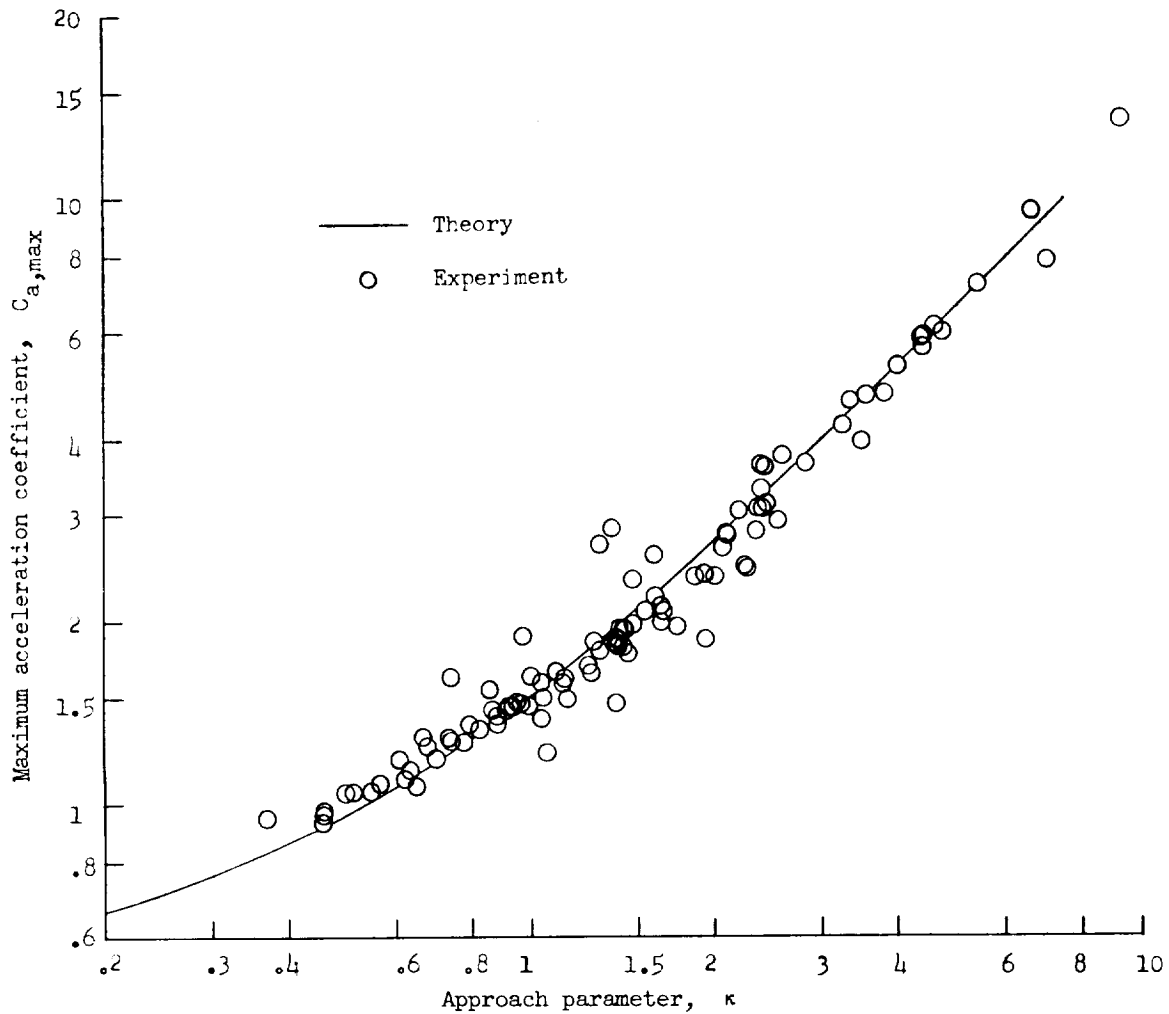


Figure 8.- Theoretical and experimental variation of maximum acceleration coefficient with approach parameter for a body having a dead-rise angle of 30° . $b = 5$ inches; $C_\Delta = 208$ to 530 ; $\tau = 6^\circ$ to 30° ; $W = 933$ to $2,350$ pounds; $\gamma_o = 2.22^\circ$ to 20.91 ; $V_o = 34.7$ to 87.8 fps.

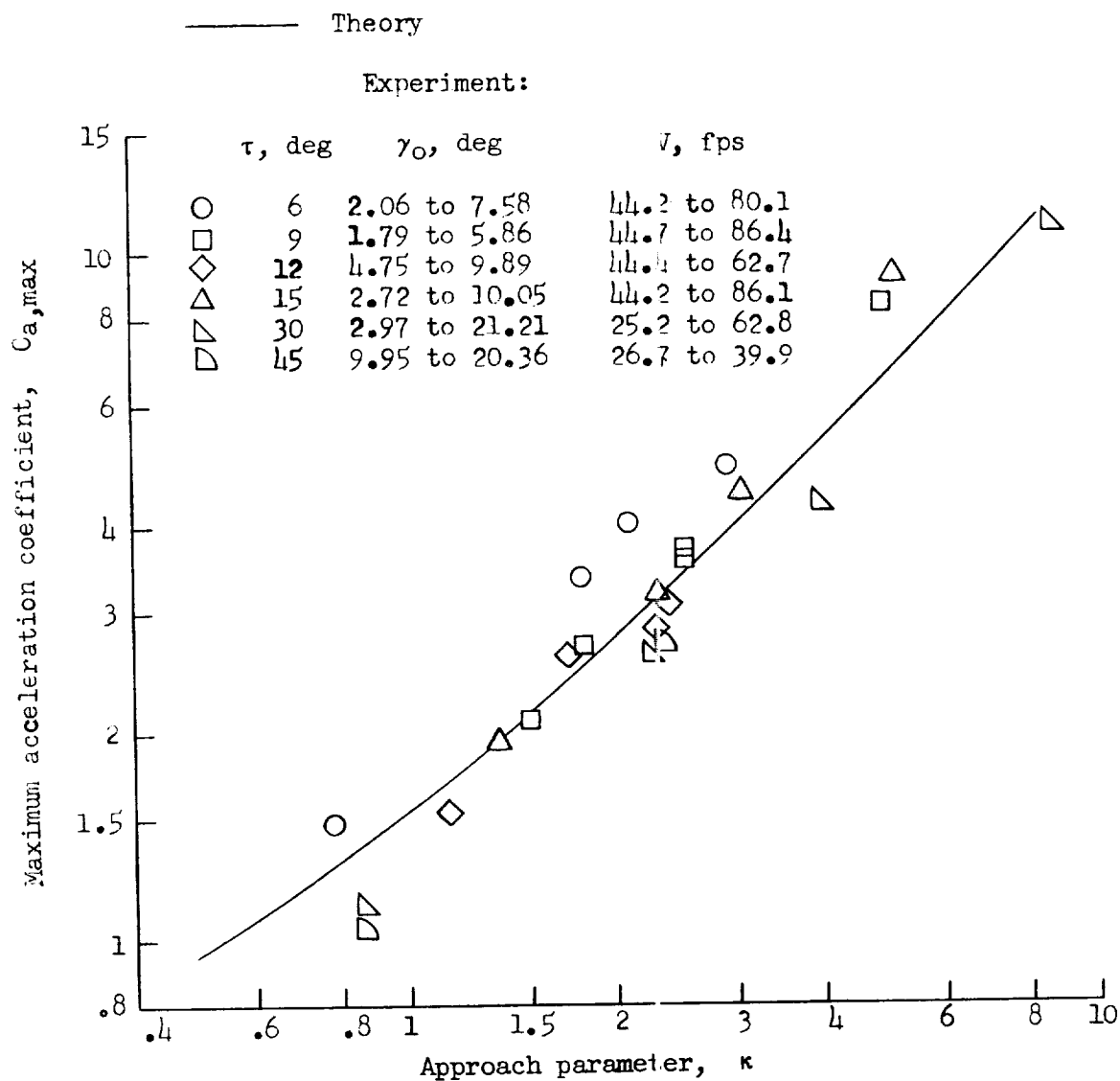


Figure 9.- Theoretical and experimental variation of maximum acceleration coefficient with approach parameter for $C_{\Delta} = 18.8$. $\beta = 0^{\circ}$; $b = 12$ inches; $W = 1,176$ pounds.

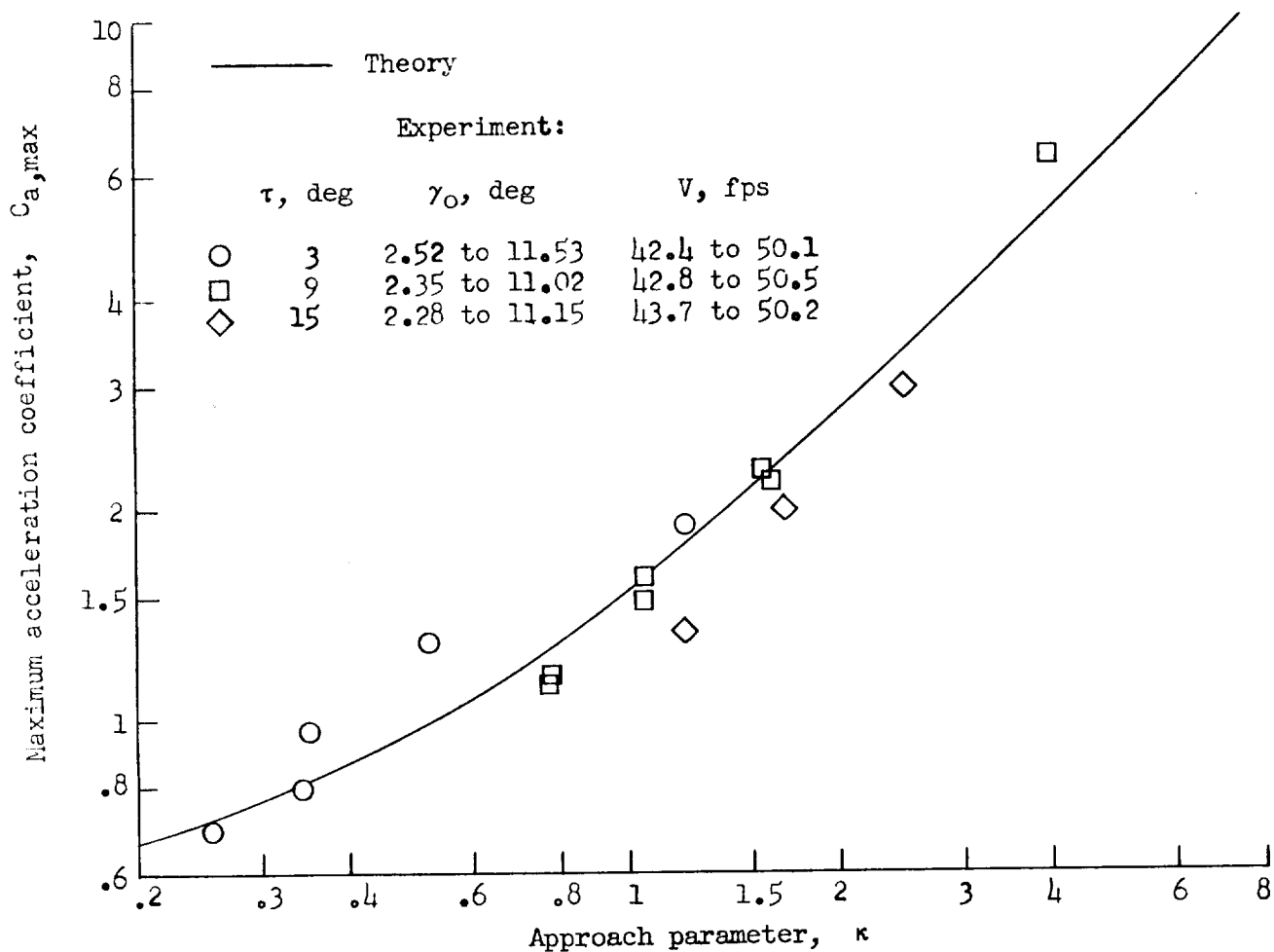


Figure 10.- Theoretical and experimental variation of maximum acceleration coefficient with approach parameter for $C_{\Delta} = 4.4$. $\beta = 0^\circ$; $b = 20$ inches; $W = 1,261$ pounds.

